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Radiative Decay of the η-Particle

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We have estimated the branching ratio of the radiative decay mode to the tripion decay mode of the η-particle

$$\eta^0 \to \pi^0 + \gamma \tag{1}$$

$$\to \pi^+ + \pi^- + \pi^0 . \tag{2}$$

The η-particle[1)] is the three-pion resonance state with $J=1^-$, $T=0$, mass ~ 550 Mev, half width ~ 25 Mev. It has been pointed out[2)] that, because of the smallness of the Q-value in the decay mode (2), the channel (1) might become relatively important and that would help the detection of the η-particle.

The phase space volumes J_1, J_2 of the modes (1) and (2) are respectively

$$J_1 = \int \delta^3(\boldsymbol{p}_1 + \boldsymbol{p}_2)\delta(m - \varepsilon_1 - \varepsilon_2) d^3\boldsymbol{p}_1 d^3\boldsymbol{p}_2 = 4\pi\mu^2 \times 1.87 , \tag{3}$$

$$J_2 = \int \delta^3(\boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p}_3)\delta(m - \varepsilon_1 - \varepsilon_2 - \varepsilon_3) \times d^3\boldsymbol{p}_1 d^3\boldsymbol{p}_2 d^3\boldsymbol{p}_3 = (4\pi)^2 \mu^5 \times 3.04 \times 10^{-1}, \tag{4}$$

where $\boldsymbol{p}_i$, ε_i are the 3-momentum and the energy of the i-th outgoing particle, $m = 4\mu$ is the mass of the η-particle, and μ is the pion mass.

Fermi's statistical theory[3)] gives a rough estimate of the branching ratio[4)]

$$\frac{w_1}{w_2} = \frac{(2\pi)^3}{\Omega} \cdot \frac{2}{137} \cdot \frac{J_1}{J_2} , \tag{5}$$

where w_1, w_2 are the decay rate of the channel (1) and (2), Ω is the interaction volume, 2 is a statistical weight factor for the photon polarization, 1/137 is the electromagnetic coupling constant singled out from the rest of the strong interaction. Assuming Ω to be the sphere of radius β in units of the pion Compton wavelength, we obtain

$$\frac{w_1}{w_2} = 0.42\beta^3 . \tag{6}$$

The simple statistical model is not a good approximation here because of the spin of the η-particle. The available method[5),6)] to take the angular momentum conservation into account is good only for a many-particle system and not for a system of 2 or 3 particles. Instead, we shall write down the general matrix element for the decay of a vector particle on the basis of invariance and amalgamate the unknown form factors into a single phenomenological parameter β.

The decay matrix element of the mode (1) is proportional to

$$M_\gamma = \frac{1}{\sqrt{k_0 q_0 p_0}} \varepsilon^{\lambda\nu\mu\sigma} k_\lambda q_\mu \epsilon_\nu \eta_\sigma \cdot \frac{1}{\sqrt{\mu}} , \tag{7}$$

where k, q and p, are the 4-momentum of the photon pion and η-particle respectively, ϵ and η are the 4-polarization of the photon and η-particle, and $\varepsilon^{\lambda\nu\mu\sigma}$ is the 4-dimensional Levi-Civita tensor. The power of μ is chosen to make M_γ dimensionless. In the rest system of the η-particle it reduces to

$$\sum_{\eta,\epsilon} |M_\gamma|^2 = \frac{2m}{\mu} \cdot \frac{\boldsymbol{k}^2}{k_0 q_0} . \tag{8}$$

Similarly the decay matrix element of the mode (2) is proportional to

$$M_\pi = \frac{1}{\sqrt{p_0^{(1)} p_0^{(2)} p_0^{(3)} p_0}} \varepsilon^{\lambda\mu\nu\sigma} p_\lambda^{(1)} p_\mu^{(2)} p_\nu^{(3)} \eta_\sigma \cdot \frac{1}{\mu} , \tag{9}$$

where $p^{(i)}$ is the 4-momentum of the i-th pion. Again in the rest system of the η-particle it becomes

$$\sum_\eta |M_\pi|^2 = \frac{m}{\mu_2} \cdot \frac{(\boldsymbol{p}^{(1)} \times \boldsymbol{p}^{(2)})^2}{p_0^{(1)} p_0^{(2)} p_0^{(3)}} . \tag{10}$$

(Because of the momentum conservation it holds that

$|\boldsymbol{p}^{(1)}\times\boldsymbol{p}^{(2)}|=|\boldsymbol{p}^{(2)}\times\boldsymbol{p}^{(3)}|=|\boldsymbol{p}^{(3)}\times\boldsymbol{p}^{(1)}|$.)

The "modified" phase space volume I_1 I_2 are defined by

$$I_1=\int\sum_{\eta,\varepsilon}|M_r|^2\delta^3(\boldsymbol{k}+\boldsymbol{q})\delta(m-k_0-q_0)d^3\boldsymbol{k}d^3\boldsymbol{q}\,,$$

$$I_2=\int\sum_{\eta}|M_\pi|^2\delta^3(\boldsymbol{p}^{(1)}+\boldsymbol{p}^{(2)}+\boldsymbol{p}^{(3)})$$

$$\times\delta(m-p_0^{(1)}-p_0^{(2)}-p_0^{(3)})d^3\boldsymbol{p}^{(1)}d^3\boldsymbol{p}^{(2)}d^3\boldsymbol{p}^{(3)}$$

and calculated to be

$$I_1=\frac{2m}{\mu}\cdot4\pi\mu^2\times1.65\,, \tag{11}$$

$$I_2=\frac{m}{\mu^2}\cdot(4\pi)^2\mu^6\times2.92\times10^{-2}. \tag{12}$$

Then the branching ratio becomes

$$\frac{w_1}{w_2}=\frac{(2\pi)^3}{\Omega}\cdot\frac{1}{137}\cdot\frac{I_1}{I_2}=3.9\beta^3. \tag{13}$$

Although the computation is of no more than an order of magnitude estimate, we observe that the angular momentum barrier reduces the tripion decay mode appreciably, so that the radiative decay mode may even dominate the tripion mode.

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